1. 1. F: X is not specified as NP or NP-hard.
   2. F: Y is not specified to be in complete, or NP, or NP-hard.
   3. F: because X can be NP.
   4. T: You can't reduce out of NP-complete and it was specified as NP, not NP-hard.
   5. F: you can reduce P as NP.
   6. T: because all P are in NP but not all NP are in P. and you can't reduce NP into P.
   7. F: Polynomials can be solved in NP so they can reduce P to NP or NP->NP.
2. 1. T: 3-SAT and TSP are both NP-complete which means they can be reduced to one another.
   2. F: 2-SAT is P but by saying P!=NP, then the NP-complete 3-SAT which is in NP can’t be reduced to a P.
   3. T: Because all P are in NP and so assuming there is a way to reduce TSP to P, then NP-complete can be reduced to NP and P.
3. Ham-Path is NP-complete:
   1. Show that HAM-path belongs to NP: Given an instance of the problem, the certificate is the sequence of n vertices in the graph G. The certifier (verification algorithm) checks that this sequence contains each vertex exactly once. This process can be done in polynomial time. Therefore HAM-path is in NP, O(V^2).
   2. Prove that HAM-path is NP-hard: We can show that HAM-cycle can reduce to HAM-path, where HAM-cycle ∈ NP-complete. Let G=(V,E) be an instance of HAM-cycle. We construct an instance of HAM-path as follows; Form the complete graph G’=(V,E’) where E’={(i,j):i,j ∈ V and i!=j} and define the cost function c by c(i,j)={0 if (i,j) ∈ E, 1 if (i,j) !∈ E, 1 for a single edge in the cycle}. The instance of HAM-path is then <G’,c,0> which is easily formed in polynomial time. By proving HAM-path-decision is NP-hard. Since HAM-path also belongs to NP, then we have shown that HAM-path-decision is NP-complete.
   3. Suppose the graph G has a HAM-cycle h. Each edge in h belongs to E with one edge added from the end vertex to the beginning vertex and thus has a cost 0 in G’. Thus h is a tour in G’ with cost 0. Conversely suppose that graph G’ has a tour h’ of cost at most 0. Since the cost of edges in E’ are 0 and 1, the most of tour h’ is exactly 0 and each edge on the tour must have cost 0. Thus h’ contains only edges in E+1. Hence we conclude that h’ is a HAM-cycle in graph G.
4. 4-color is NP-complete:
   1. Show that 4-color belongs to NP: Given an instance of the problem the certificate is the number n colors (vertices). The certifier checks that each vertex has an edge to another vertex of a different color. This process can be done in polynomial time. Therefore 4-color-decision in in NP.
   2. Prove that 4-color is NP-hard: We can show that 3-color reduces to 4-color for some 3-color ∈ NP-complete. Let G=(V,E) be an instance of 3-color. We construct an instance of 4-color as follows. Form the complete graph G’=(V,E) where c:V->{1,2,3} such that c(u) != c(v) for every edge (u,v) ∈ E. Then add 4 to c:V making it c:V->{1,2,3,4}. This new vertex is of a new color and connects to each other vertex. The instance of 4-color is then <G’,c,4> which is easily formed in polynomial time. By proving 4-color is NP-hard. Since 4-color is proven as NP, then we have shown that 4-color-decision is NP-complete.